Problem Set #3

1) The full-wave rectifier circuit shown below has an input signal whose frequency is 60 Hz. The rms value of $v_s$ is 8.5 V. Assume each diode turn-on voltages is $V_D = 0.7$ V.

(a) What is the maximum value of $V_D$.

(b) If $R = 10 \Omega$, determine the value of $C$ such that the ripple voltage is no larger than 0.25 V.

(c) What must be the PIV rating of each diode?

2) Assuming the turn-on voltage $V_D = 0.7$ V, describe and plot the transfer characteristic of the circuit shown below.

3) The circuit shown below is a voltage-doubler. The input is a sinusoidal of 10 V peak and 1 kHz frequency. Assume that the diodes have parameters of $I_S = 2.682 \text{nA}$, $n = 1.836$, $R_S = 0.5664 \Omega$, $V_0 = 0.5$ V, $C_{j0} = 4 \text{ pF}$, $m = 0.333$, $\tau_T = 11.54 \text{ ns}$, $V_{ZK} = 100$ V, $I_{ZK} = 100 \mu\text{A}$. $C_1 = C_2 = 1 \mu\text{F}$.

(a) Find the steady-state output voltage of the circuit.

(b) Use HSPICE to plot the transient behavior of the voltages across $D_1$ and $C_2$.

4) For a pn junction diode, the acceptor concentration is $N_A = 2 \times 10^{16} \text{ cm}^{-3}$ and the donor concentration is $N_D = 2 \times 10^{15} \text{ cm}^{-3}$. Assume $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$ and the junction area is $A = 400 \mu\text{m}^2$.

(a) Find the junction built-in voltage $V_0$;

(b) Find the width of the depletion region, $W_{dep}$, and its extent in each of the p and n regions when the junction is reverse biased with $V_R = 5$ V;

(c) At $V_R = 5$ V, calculate the magnitude of the charge stored on either side of the junction;

(d) Calculate the total junction capacitance $C_j$.

5) (a) In a forward-biased pn junction show that the ratio of the current component due to hole injection across the junction to the component due to electron injection is given by $\frac{I_p}{I_n} = \frac{D_p}{D_n} \frac{L_n}{L_p} \frac{N_A}{N_D}$.
(b) A *p*–*n* diode is one in which the doping concentration in *p* region is much greater than in the *n* region. In such a diode, the forward current is mostly due to hole injection across the junction.

Show that \( I = I_n + I_p \equiv I_p = Aq n_i^2 \frac{D_p}{L_p N_D} \left[ \exp \left( \frac{V}{V_T} \right) - 1 \right] \).

(c) For the specific case in which \( N_D = 5 \times 10^{16} \text{ cm}^{-3} \), \( D_p = 10 \text{ cm}^2/\text{s} \), \( \tau_p = 0.1 \mu \text{s} \), and \( A = 10^4 \mu \text{m}^2 \), find \( I_S \) and the voltage \( V \) obtained when \( I = 0.2 \text{mA} \). Assume operation at 300 K where \( n_i = 1.5 \times 10^{10} \text{ cm}^{-3} \). Also, calculate the excess minority-carrier charge and the value of the diffusion capacitance at \( I = 0.2 \text{mA} \).

**Solution to Problem Set #3**

1) (a) \( V_p = \sqrt{2} v_p (\text{rms}) = \sqrt{2} \times 8.5 = 12.02 \text{ V} \). \( V_o = v_o (\text{max}) = V_p - V_D = 12.02 - 0.7 = 11.32 \text{ V} \).

2) \( V_o \leq -5.7 \text{ V} \rightarrow \frac{v_i - v_o}{10} = \frac{v_o + 5.7}{10} \rightarrow v_o = \frac{1}{2} v_i - 2.85 \rightarrow v_i \leq -5.7 \text{ V} \).

\(-5.7 \text{ V} \leq v_o \leq 5.7 \text{ V} \rightarrow v_o = v_i \rightarrow -5.7 \text{ V} \leq v_i \leq 5.7 \text{ V} , \)

\( v_o \geq 5.7 \text{ V} \rightarrow \frac{v_i - v_o}{10} = \frac{v_o - 5.7}{10} \rightarrow v_o = \frac{1}{2} v_i + 2.85 \rightarrow v_i \geq 5.7 \text{ V} . \)

3) (a) \( v_{p1} = -V_p + V_p \sin \omega t \), \( v_o = v_{p2} + v_{p1} = -V_p + V_p \sin \omega t \). In steady state, \( v_o = -2V_p = -20 \text{ V} \).
The netlist is
Title Exercise 3.35 - Voltage Doubler
* Circuit Description
Vs in 1 0 sin(0 10V 1kHz)
C1 1 2 1u
C2 3 0 1u
D1 2 0 D1N4148
D2 3 2 D1N4148
.model D1N4148 D (Is=2.682nA Rs=0.5664 Vj=0.5V n=1.836 + Cj0=4pF Tt=11.54ns m=0.333 Bv=100 Ibv=100uA)
* Analysis Requests
.Tran 0.01ms 10ms
* Output Requests
* Print the magnitude and phase of the output voltage
* as a function of frequency
.plot tran V(1) V(2) V(3)
.end

4) (a) \( V_0 = V_T \ln \left( \frac{N_A N_D}{n_t^2} \right) = 0.025 \times \ln \left[ \frac{2 \times 10^{15} \times 2 \times 10^{16}}{(1.5 \times 10^3)^2} \right] = 0.648 \text{V} \)

(b) \( W_{dep} = x_n + x_p = \sqrt{\frac{2 \varepsilon_{5}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right)(V_0 + V_R)} \),

\[
W_{dep} = \sqrt{\frac{2 \times 11.7 \times 8.854 \times 10^{-14}}{1.6 \times 10^{-39}}} \left( \frac{1}{2 \times 10^{15}} + \frac{1}{2 \times 10^{16}} \right)(0.648 + 5) = 2.0 \times 10^{-4} \text{cm} = 2.0 \mu \text{m}.
\]

\( N_D x_n = N_A x_p \rightarrow x_n = \frac{N_A}{N_D} x_p = 10x_p \rightarrow W_{dep} = 11x_p, \quad x_p = \frac{2.0}{11} = 0.182 \mu \text{m} \rightarrow x_n = 1.818 \mu \text{m} \).

(c) \( Q_N = qAN_D^N x_n = qAN_D^N \frac{N_A}{N_D + N_A} W_{dep} \).
\[ Q_n = 1.6 \times 10^{-19} \times 400 \times 10^{-8} \times \frac{2 \times 10^{15} \times 2 \times 10^{16}}{2 \times 10^{15} + 2 \times 10^{16}} \times 2.0 \times 10^{-4} = 2.328 \times 10^{-13} \text{C}, \]

\[ |Q_p| = qA N_A x_p = qA N_A \frac{N_D}{N_D + N_A} W_{dep}, \]

\[ |Q_p| = 1.6 \times 10^{-19} \times 400 \times 10^{-8} \times \frac{2 \times 10^{15} \times 2 \times 10^{16}}{2 \times 10^{15} + 2 \times 10^{16}} \times 2.0 \times 10^{-4} = 2.328 \times 10^{-13} \text{C} \]

\[ C = \frac{A \varepsilon_s}{W_{dep}} = \frac{400 \times 10^{-8} \times 11.7 \times 8.854 \times 10^{-14}}{2.0 \times 10^{-4}} = 2.072 \times 10^{-14} \text{F} = 20.72 \text{fF} \]

5) (a) \[ I_p = A q n_i^2 \left( \frac{D_p}{L_p N_D} \right) \left[ \exp \left( \frac{V}{V_T} \right) - 1 \right], \]
\[ I_n = A q n_i^2 \left( \frac{D_n}{L_n N_A} \right) \left[ \exp \left( \frac{V}{V_T} \right) - 1 \right], \]
\[ I_p = D_p L_n N_A \frac{I_n}{I_p} = D_p L_n N_A. \]

(b) \[ N_A \gg N_D \rightarrow I_p >> I_n \rightarrow I = I_n + I_p \cong I_p = A q n_i^2 \frac{D_p}{L_p N_D} \left[ \exp \left( \frac{V}{V_T} \right) - 1 \right], \]

(c) \[ L_p = \sqrt{\frac{D_p \tau_p}{\tau_p}} = \sqrt{10 \times 0.1 \times 10^{-6}} = 10^{-3} \text{cm}, \]
\[ I = A q n_i^2 \frac{D_p}{L_p N_D} \left[ \exp \left( \frac{V}{V_T} \right) - 1 \right], \]
\[ I = 10^4 \times 10^{-8} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2 \times \frac{10}{10^{-3} \times 5 \times 10^{16}} \times \left[ \exp \left( \frac{V}{0.025} \right) - 1 \right] = 0.2 \times 10^{-3}, \]
\[ V = 0.684 \text{V}. \] Excess minority charge stored is \[ Q = Q_n + Q_p = \tau_p I_p + \tau_n I_n \cong \tau_p I_p, \]
\[ Q \cong 0.1 \times 10^{-6} \times 0.2 \times 10^{-3} = 2 \times 10^{-11} \text{C}, \]
\[ C_d = \frac{\tau_p I_p}{V_T} = \frac{2 \times 10^{-11}}{0.025} = 8 \times 10^{-10} \text{F} = 800 \text{pF} \]