4.3 Derivative and the Shapes of Graphs

- What does $f'$ say about $f$?

**Def.** (a) If $f'(x) > 0$ on an interval, then $f$ is *increasing* on that interval.

(b) If $f'(x) < 0$ on an interval, then $f$ is *decreasing* on that interval.

**Exp.** Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and where it is decreasing.
The First Derivative Test Suppose that $c$ is a critical number of a continuous function $f$, where $c \in (a, b)$.

(a) If $f'(x) > 0 \forall x \in (a, c)$ and $f'(x) < 0 \forall x \in (c, b)$, then $f$ has a local maximum at $c$.

(b) If $f'(x) < 0 \forall x \in (a, c)$ and $f'(x) > 0 \forall x \in (c, b)$, then $f$ has a local minimum at $c$.

(b) If $f'$ does not change sign at $c$, then $f$ has no local maximum or minimum at $c$. 
(a) Local maximum

(b) Local minimum

(c) No maximum or minimum
Exp. \( f(x) = 3x^4 - 4x^3 - 12x^2 + 5 \), find local minimum and local maximum of \( f \).

- **What does \( f'' \) say about \( f \)?**

**Def.**
(a) If the graph of \( f \) lies above all of its tangents on \( I \), then it is called **concave upward** on \( I \).
(b) If the graph of \( f \) lies below all of its tangents on \( I \), then it is called **concave downward** on \( I \).
Concavity Test

(a) If \( f''(x) > 0 \ \forall x \in I \), then \( f \) is concave upward on \( I \).

(b) If \( f''(x) < 0 \ \forall x \in I \), then \( f \) is concave downward on \( I \).
Def. A point \( P \) on a curve \( y = f(x) \) is called an inflection point (反曲點) if \( f \) is continuous there and the curve changes the concavity at \( P \).
The Second Derivatives Test  Suppose $f''$ is continuous near $C$. 

(a) If $f'(c) = 0$ and $f''(c) > 0$, then $f$ has a local minimum at $C$.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then $f$ has a local maximum at $C$. 
Exp. Sketch a possible graph of a function $f$ that satisfies the following conditions:

1. $f'(x) > 0$ on $(-\infty, 1)$, $f'(x) < 0$ on $(1, \infty)$.
2. $f''(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$, $f''(x) < 0$ on $(-2, 2)$.
3. $\lim_{x \to -\infty} f(x) = -2$, $\lim_{x \to \infty} f(x) = 0$. 
Exercise 4.3

1, 2, 4, 5, 6, 8, 11, 12, 13, 14, 15, 16, 17, 21, 22, 23, 24, 25, 27, 28, 29, 30, 43, 49, 57.