### 4.7 Antiderivatives

**Def.** A function $F$ is called an antiderivative of $f$ on an interval $I$ if $F'(x) = f(x)$ for all $x \in I$.

**Theorem.** If $F$ is an antiderivative of $f$ on $I$, then $F(x) + c$ is antiderivative of $f$. 
Exp. Find antiderivative of each of the following functions.

(a) \( f(x) = \sin x \)

(b) \( f(x) = \frac{1}{x} \)

(c) \( f(x) = x^n; n \neq -1 \)

(d) \( f(x) = e^x \)

(e) \( f(x) = \cos x \)

(f) \( f(x) = \sin x \)

(g) \( f(x) = \sec^2 x \)

(h) \( f(x) \sec x \tan x \)

(i) \( f(x) = \frac{1}{1 + x^2} \)

(j) \( f(x) = \frac{1}{\sqrt{1 - x^2}} \)
Theorem. If \( F (x) \) and \( G (x) \) are the antiderivatives of \( f (x) \) and \( g (x) \) respecting, then

1. \( cF (x) \) is an antiderivative of \( cf (x) \)
2. \( F (x) \pm G (x) \) is an antiderivative of \( f (x) \pm g (x) \)

Exp. Find \( f \) if

\[
f'(x) = 4 \sin x + \frac{2x^5 - \sqrt{x}}{x}
\]

Exp. Find \( f \) if \( f'(x) = e^x + 20 \left( 1 + x^2 \right)^{-1} \) and \( f(0) = -2 \)
Exp. Find \( f \) if \( f''(x) = 12x^2 + 6x - 4 \),
\[
f(0) = 4 \quad \text{and} \quad f(1) = 1.
\]

Exercise 4.7
1, 3, 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, 22, 25, 27, 28, 31.